

5. Decimals

Things you might need to be able to do with decimals...

This will vary with your age and what maths set you are in, but here is a list of some of the things you might need to be able to do with decimals:

1. Know the three types of decimals
2. Know how to add, subtract, multiply and divide using decimals
3. Understand the relationship between fractions, decimals and percentages
4. Know how to convert a recurring decimal into a fraction.

And if you are sitting comfortably, then I will begin...

1. The Three Types of Decimal

Here they are...

(a) Exact or Terminating

These are decimals that stop.

They do not go on forever, and so you can write down all their digits

e.g. 0.5, 0.276, 0.523894, 0.0000000004



(b) Recurring

These are decimals that **go on forever and ever, but some of the digits repeat.**

e.g. 0.3333333... 0.454545454545..., 0.1489148914891489...,



It is impossible to write down all the digits, so we use the dot notation

We place a dot (or two dots) to show which digits repeat

Examples

0.5555555555...	Here, only the 5 repeats, so we place the dot like this:	→	0.5̇
0.1431431431...	Now it's the 143 repeating, so the following is needed:	→	0.1̇43̇
0.3777777777...	The 7 repeats, but the 3 does not, so it would be wrong to put a dot on the 3. We need this:	→	0.37̇
0.82341341341...	Here, only the 341 is repeating, so place the dots at the beginning and end of that group	→	0.823̇41̇

(c) Irrational

These are what I call dodgy decimals

They go on forever and ever, but the digits do not repeat in a regular pattern

e.g. pi, which is 3.1415926535897932...

Watch Out: Your calculator only has so much space, so some recurring or irrational decimals might look like the actually terminate!

2. Working with Decimals

If you can add, subtract, multiply and divide whole numbers (integers), then you should be okay with decimals, so long as you are careful!

(a) Adding and Subtracting

The key thing here is to write the numbers out on top of each other and **line up your decimal points**. That way you won't make a daft mistake

Example 1

$$12.875 + 0.34$$

Write the numbers out on top of each other, and line up your decimal points

$$\begin{array}{r} 12.875 \\ + 0.34 \\ \hline \end{array}$$

Now, so long as you remember how to add, and be careful when **carrying numbers**, you should get the answer of:

$$13.215$$



Example 2

$$0.62 - 0.0159$$

Again, write the numbers out on top of each other, and line up your decimal points

$$\begin{array}{r} 0.6200 \\ - 0.0159 \\ \hline \end{array}$$

Sometimes I find it easier to fill in **zeros** on the top line to make subtracting easier.

and remember all your rules from primary school about **borrowing from the number to the left**

You should get:

$$0.6041$$

(b) Multiplying

There are a few different ways of doing this, so feel free to ditch my method if you find a better one, but basically if there is a **whole number involved** I do the sum as normal, and if there are **two decimals**, then I change the question to make it easier!

Example 1

$$7 \times 1.36$$

Write the numbers out on top of each other, putting the **whole number on the bottom**

Remember: it doesn't matter which order you do multiplications

$$\begin{array}{r} 1.36 \\ \times \quad 7 \\ \hline \end{array}$$

Now, just multiply each digit in turn **by 7**, and remember to **carry your numbers**, and you should end up with:

$$9.52$$



Example 2

$$0.32 \times 0.528$$

Now, I don't like the look of this at all, but if I multiply the **0.32 by 100** and the **0.528 by 1000**, then I get a much easier sum:

$$32 \times 528$$

Now, I do these kind of sums using the grid method, but feel free to use your own way:

	500	20	8	
30	15000	600	240	15000 1000 600 240 40 16 <hr/> 16896
2	1000	40	16	

Now, the answer to this question is 16,896, but to get the answer to the original question we must **undo our changes**, so **divide by 100** and then **divide by 1000**

Which gives us: **0.16896**

(c) Dividing

Fortunately, questions about **dividing with decimals** do not come up all that often, but whenever they do I again use the same method:

1. Make the question **easier** by **multiplying** the numbers by 10, 100, or 1000
2. Do the easier sum
3. Remember to **undo your changes** by multiplying **or** dividing by 10, 100, or 1000 to get the actual answer

Example $75.92 \div 1.3$



Right, let's sort those horrible numbers out first.

Multiply the 75.92 by 100 and the 1.3 by 10, and things \longrightarrow $7592 \div 13$
should look a whole lot nicer

Now, it all depends how you like to do these. I write it out like this:

$$13 \overline{)7592}$$

And then I do a lot of **talking to myself** like this:

- How many 13s go into 7?... None, so carry the seven
- How many 13s go into 75?... Erm... 5, remainder 10, so carry the 10
- How many 13s go into 109?... Erm... erm... 8, remainder 5
- How many 13s go into 52?... 4 exactly!

$$13 \overline{)7592} \begin{array}{r} 0584 \end{array}$$

So, our answer is **584**, but remember we must undo our changes

Well, multiplying 75.92 by 100 made the answer **100 times too big**, so we must **divide by 100**, **BUT:** multiplying 1.3 by 10 made the answer **10 times too small** (as we were dividing), so we must **multiply by 10**

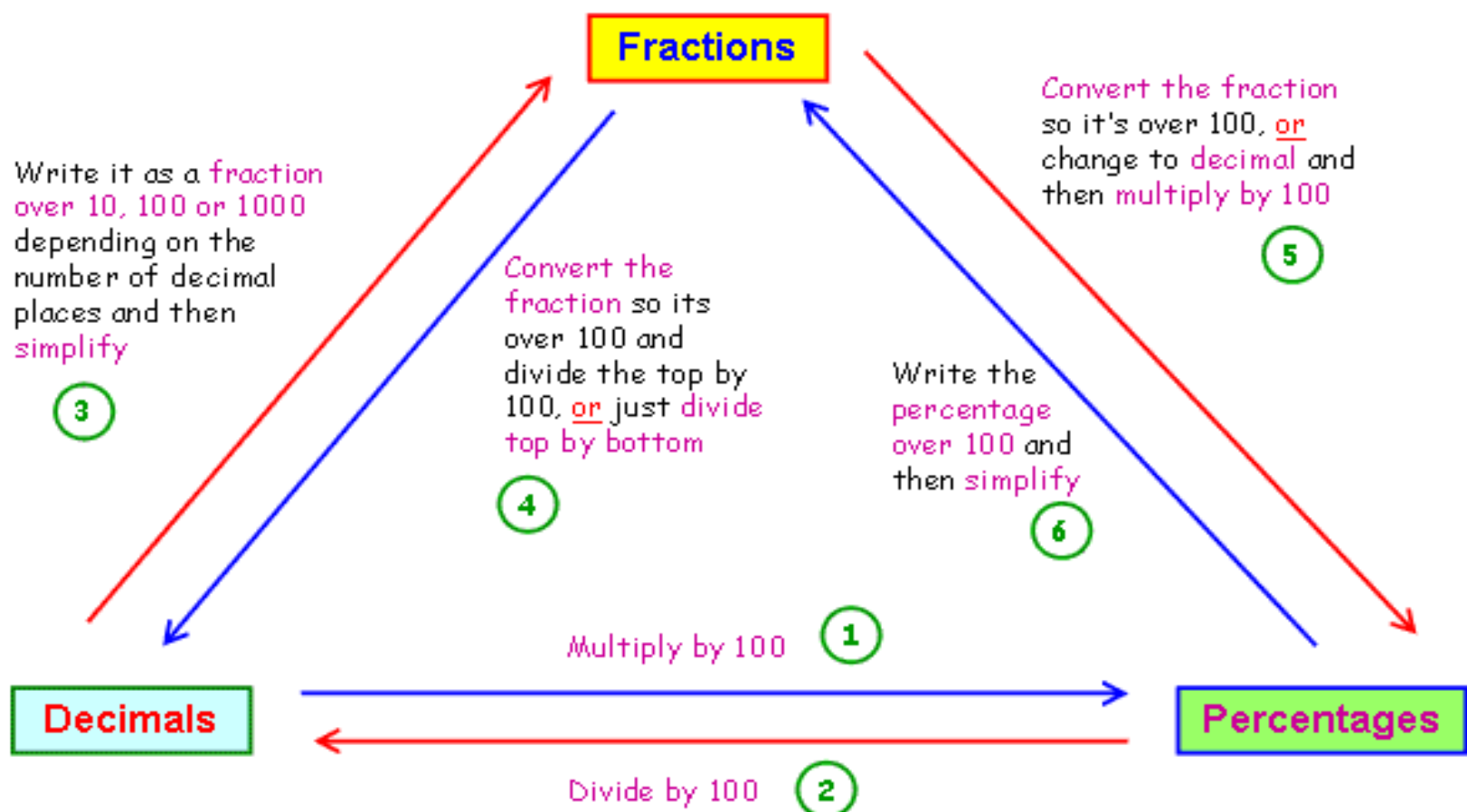
Which gives us: \longrightarrow **58.4**

3. Fractions, Decimals and Percentages

Fractions, Decimals and Percentages are all closely related to each other, and you need to be comfortable changing between each of them.

Hopefully this diagram will help.

Follow the arrows depending on what you need to change, and follow the numbers for examples below



1 What is 0.364 as a percentage?

Just multiply by 100 and be careful with the decimal point!

$$0.364 \times 100 = 36.4\%$$

2 Convert 8.3% into a decimal

Just divide by 100 and again be careful with the decimal point!

$$8.3 \div 100 = 0.083$$

3 Write 0.16 as a fraction

There are 2 decimal places, so write it over 100

$$\frac{16}{100}$$

Now carefully simplify

$$\frac{16}{100} = \frac{8}{50} = \frac{4}{25}$$

4 Write $\frac{13}{20}$ as a decimal

We need to change the bottom of the fraction to 100, remembering to do the same to the top

$$\frac{13}{20} \overset{\times 5}{=} \frac{65}{100} \underset{\times 5}{} = 0.65$$

Divide the top of your fraction by 100 and you have your answer!

= 0.65

5 Write $\frac{5}{8}$ as a percentage

It's not easy to change this fraction over 100, so we must divide 5 by 8

$$5 \div 8$$

Use any method, but I do this:

$$= 8 \overline{) 5.000} \begin{array}{r} 0.625 \end{array}$$

0.625 is the answer as a decimal, so we must multiply by 100

$$0.625 \times 100 = 62.5\%$$

6 What is 12.5% as a fraction?

Start by writing the percentage over 100

$$\frac{12.5}{100}$$

We need to simplify, but the decimal point makes it hard. So why not multiply top and bottom by 2!

$$\times 2 \quad \frac{25}{200}$$

Now we can simplify as normal to get the answer:

$$\frac{25}{200} = \frac{5}{40} = \frac{1}{8}$$

4. Convert a Recurring Decimal into a Fraction

As I hope you've seen, its not too bad to convert **fractions into decimals**, or **terminating decimals into fractions**, but what about **recurring decimals** that go on forever?

Warning: This is hard! But bare with me and I'll try to take you through an example:

Example Convert 0.165165165165... into a fraction



1. We start with a bit of **algebra**:

$$\text{Let } x = 0.165165165165\dots$$

2. We want to move the **decimal point to the right** so that the repeated block of digits appears **in front of the decimal point**, so... to move the point 3 places we must **multiply by 1000!**

$$0.165165165165\dots \times 1000 = 165.165165165\dots$$

3. Now, if $0.165165165\dots = x$, then $165.165165165\dots$ must equal... **$1000x$!**

$$1000x = 165.165165165165\dots$$

$$x = 0.165165165165\dots$$

4. This is the **clever/hard** bit. If we **subtract** the top from the bottom we get:

$$999x = 165$$

$$1000x - x = 999x$$

$$165.165165165 - 0.165165165 = 165$$

5. So, by **dividing both sides by 999** we get:

$$x = \frac{165}{999}$$

Which, if you check on a calculator is correct!

Challenge: When you convert some fractions into decimals, such as $\frac{7}{8}$ $\frac{3}{10}$ $\frac{9}{16}$ $\frac{11}{50}$ you get a **terminating decimal**, but with others such as $\frac{2}{3}$ $\frac{6}{11}$ $\frac{5}{12}$ $\frac{3}{20}$ $\frac{8}{41}$ You get a **recurring decimal!** What is the rule?