# 6. Fractions

## Things you might need to be able to do with fractions...

This will vary with your age and what maths set you are in, but here is a list of some of the things you might need to be able to do with fractions:

1. Know the all important fraction's lingo
2. Know how to find the fraction of a quantity
3. Know all about equivalent fractions so you can simplify
4. Understand proper and improper fractions
5. Know how to add, subtract, multiply and divide using decimals
6. Understand the relationship between fractions, decimals and percentages

And so, without further ado, let's get on with it...

## 1. Fraction's Lingo

Right, let's try and stop talking about the top and the bottom of a fraction, and instead go for something a bit fancier, like this:

$$\frac{3}{4}$$

numerator

denominator

Warning: every now and again, Mr Barton forgets to call them this, so watch out for "top" and "bottom" creeping into these notes

## 2. Fraction of a Quantity

There is a simple method which always works with these types of questions:

**1.** Divide by the bottom (finds you the value of one fraction)
**2.** Multiply by the top (gives you the value of the number of fractions you need)

### Example 1

What is $\frac{3}{4}$ of 24?

**1.** If you divide the quantity (24) by the denominator of the fraction (4), it tells you the value of $\frac{1}{4}$

$24 \div 4 = 6$ so, $\frac{1}{4} = 6$

**2.** But we don't want $\frac{1}{4}$, we want $\frac{3}{4}$

so, we must multiply our answer (6) by the numerator (3)

$6 \times 3 = 18$ so, $\frac{3}{4} = 18$

### Example 2

Find $\frac{5}{7}$ of 2.436 Kg
(give your answer in grams)

Now, if before we start, let's change 2.436 Kg into grams so we get nicer numbers and so we are ready for our answer:

$2.436 \times 1000 = 2436$ so, 2.436kg = 2436g

Now we just do the same as before:

**1.** Divide by the bottom (2436 ÷ 7)

**2.** Multiply our answer by the top ( × 5 )

$\frac{1}{7} = 348$ so, $\frac{5}{7} = 1740$

Remember: give units in your answer:

**1740g**

## 3. Equivalent Fractions

Equivalent fractions are just fractions which have exactly the same value.

You need good knowledge of equivalent fractions when simplifying your answers, and also when adding and subtracting fractions.

Here is the rule:

Whatever you multiply or divide the top by, do the exact same to the bottom!

### Example 1

$$\frac{2}{7} = \frac{?}{21}$$

Ask yourself: "what has been done to the 7 to make it 21?"

And then do the same to the top!

$$\frac{2}{7} \overset{\times 3}{=} \frac{6}{21}$$

### Example 2

$$\frac{49}{70} = \frac{7}{?}$$

Ask yourself: "what has been done to the 49 to make it 7?"

And then do the same to the bottom!

$$\frac{49}{70} \overset{\div 7}{=} \frac{7}{10}$$

### Example 3

Simplify: $\frac{48}{54}$

We are looking to make the fraction as simple as possible (i.e. contain the smallest possible whole numbers)

We need a number to divide both the top and the bottom by (a factor of both)

We stop dividing when the top and the bottom do not share any more factors

It doesn't matter how long it takes!

$$\frac{48}{54} \overset{\div 2}{=} \frac{24}{27} \overset{\div 3}{=} \frac{8}{9}$$

## 4. Proper and Improper Fractions

In a proper fraction, the bottom is bigger than the top, like: $\frac{3}{4}$

In an improper fraction (top heavy), the top is bigger than the bottom, like: $\frac{9}{7}$

Sometimes, improper fractions are written as mixed number fractions, like: $3\frac{2}{7}$

You need to be able to switch between improper and mixed number fractions!

### Example 1

Write: $22/5$ as a mixed number fraction

Okay, so here we have 22 lots of $1/5$

How many $1/5$ do we need to make one whole?

Well, if you think about a cake sliced into fifths, then we would need 5 slices to make a whole

So, how many wholes can we make out of our 22?

Well, 5 goes into 22... erm... 4 times, with a remainder of... erm... erm... 2!

So, our 22 makes 5 wholes with 2 parts left over

So... $\frac{22}{5} = 4\frac{2}{5}$

### Example 2

Write: $3\frac{5}{8}$ as an improper fraction

Right, now we have 3 whole ones, and 5 lots of $1/8$

How many lots of $1/8$ are then in each whole?...

Well, one whole is $8/8$, so there must be 8!

So, how many lots of $1/8$ in our 3 wholes?

$3 \times 8 = 24$!

But remember, we also have our 5 lots of $1/8$

So, altogether we have (24 + 5) lots of $1/8$

So... $3\frac{5}{8} = \frac{29}{8}$

## 5. Adding, Subtracting, Multiplying and Dividing Fractions

Warning: this is one of those topics everyone messes up!

Don't mix up your rules for adding and subtracting with those for multiplying and dividing!

(a) Adding and Subtracting

1. Change any mixed number fractions into improper (top heavy) fractions
2. Choose a number that both denominators go into (are factors of)
3. Use your skills of equivalent fractions to make both fractions have that chosen number as their denominator
4. Add/subtract the numerators together, keep the denominator the same, and simplify!

Why can't I just add the tops and the bottom together, cos that'd be dead easy?...

Imagine doing this question $\frac{1}{3}+\frac{1}{5}$

So, you want to add the tops and the bottoms... $\frac{1}{3}+\frac{1}{5}=\frac{2}{8}$

Simplify it: $\frac{2}{8}=\frac{1}{4}$

But look! We started off with $\frac{1}{3}$, we added something to it, we got $\frac{1}{4}$ for our answer, which is smaller than $\frac{1}{3}$

THIS IS ABSOLUTE RUBBISH!!!

(but people still do it!)

**Example** $3\frac{1}{3}-\frac{4}{5}$

1. Change the mixed number fraction: $3\frac{1}{3}=\frac{10}{3}$
2. Choose a number both denominators are factors of

3 and 5 are both factors of 15

3. Change both fractions so they have 15 on the bottom:

$$\frac{10}{3}\ \overset{\times 5}{=}\ \frac{50}{15} \qquad \frac{4}{5}\ \overset{\times 3}{=}\ \frac{12}{15}$$

4. Subtract tops, leave bottoms, simplify:

$$\frac{50}{15}-\frac{12}{15}=\frac{38}{15}=2\frac{8}{15}$$

(a) Multiplying and Dividing

Good News: This is a lot easier than adding and subtracting!

**How to Multiply fractions:**

(1) Change any mixed number into improper (top heavy) fractions

(2) Multiply tops together and multiply bottoms together

(3) Simplify your answer

**How to Divide fractions:**

(1) Change mixed number fractions

(1) Flip the second fraction upside down

(2) Change the division sign to a multiply

(3) Multiply and simplify!

**Example 1** $\frac{2}{5} \times 1\frac{3}{4}$

1. Change the mixed number fraction: $1\frac{3}{4} = \frac{7}{3}$

2. Multiply tops together and bottoms together:

$$\frac{2}{5} \times \frac{7}{4} = \frac{2 \times 7}{5 \times 4} = \frac{14}{20}$$

3. Simplify:

$$\frac{14}{20} = \frac{7}{10}$$

**Example 2** $3\frac{2}{7} \div \frac{5}{6}$

1. Change the mixed number fraction: $3\frac{2}{7} = \frac{23}{7}$

2. Flip the second fraction: $\frac{5}{6} \rightarrow \frac{6}{5}$

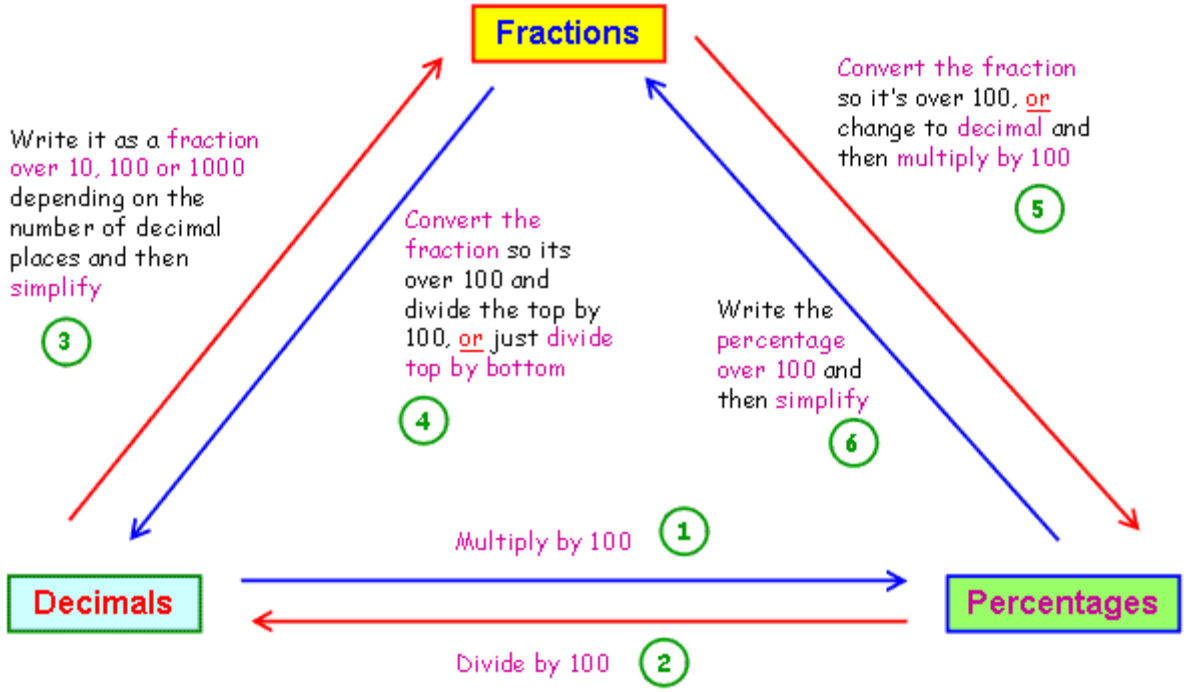
3. Change sign to multiply: $\frac{23}{7} \times \frac{6}{5}$

4. Finish it off by multiplying and then simplifying!

$$\frac{23}{7} \times \frac{6}{5} = \frac{23 \times 6}{7 \times 5} = \frac{138}{35} = 3\frac{33}{35}$$

## 5. Fractions, Decimals and Percentages

Fractions, Decimals and Percentages are all closely related to each other, and you need to be comfortable changing between each of them.
Hopefully this diagram will help.
Follow the arrows depending on what you need to change, and follow the numbers for examples below



**1** What is 0.364 as a percentage?

Just multiply by 100 and be careful with the decimal point!

$0.364 \times 100$

$= 36.4\%$

**2** Convert 8.3% into a decimal

Just divide by 100 and again be careful with the decimal point!

$8.3 \div 100$

$= 0.083$

**3** Write 0.16 as a fraction

There are 2 decimal places, so write it over 100

$$\frac{16}{100}$$

Now carefully simplify

$$\frac{16}{100} = \frac{8}{50} = \frac{4}{25}$$

**4** Write $\frac{13}{20}$ as a decimal

We need to change the bottom of the fraction to 100, remembering to do the same to the top

$$\frac{13}{20} \overset{\times 5}{=} \frac{65}{100}$$

Divide the top of your fraction by 100 and you have your answer!

$= 0.65$

**5** Write $\frac{5}{8}$ as a percentage

It's not easy to change this fraction over 100, so we must divide 5 by 8

$5 \div 8$

Use any method, but I do this:

$$= 8\overline{)5.000}\ 0.625$$

0.625 is the answer as a decimal, so we must multiply by 100

$0.625 \times 100$

$= 62.5\%$

**6** What is 12.5% as a fraction?

Start by writing the percentage over 100

$$\frac{12.5}{100}$$

We need to simplify, but the decimal point makes it hard. So why not multiply top and bottom by 2!

$\times 2$ $\frac{25}{200}$

Now we can simplify as normal to get the answer:

$$\frac{25}{200} = \frac{5}{40} = \frac{1}{8}$$