7. Percentages

Things you might need to be able to do with percentages...

This will vary with your age and what maths set you are in, but here is a list of some of the things you might need to be able to do with percentages:

1. Understand what percentages are
2. Find the percentage of an amount in your head and with a calculator
3. Calculate percentage change
4. Calculate percentage increase
5. Calculate percentage decrease
6. Understand compound interest
7. Be able to calculate reverse percentages

It's a big one this one, so without further ado, let's get going...

1. What are percentages?

A percentage is just a fraction whose denominator (bottom) is 100.

So, if we say "32%", what we mean is $\frac{32}{100}$

Now, what is really important before we start is that you understand how to change back and forward from percentages to fractions and decimals.

e.g. You need to able to say that if someone got 23 out of 45 in a test, this is the same as getting 51.1% (1dp)

It's probably best to go back to 6. Fractions, or 5. Decimals, until you are happy with this

2. Percentage of an Amount

For easy numbers, these sort of questions are fine to do armed merely with a pencil, some paper, and your brain. However, you also need to be able to them on your calculator.

(a) In Your Head

With a bit of knowledge about dividing things by 10 and 100, and some common sense, it's not too bad working out percentage questions in your head.

Example You have £320. What is (a) 15%, (b) 63%, (c) 17.5%

I always start these by writing down the percentages that I know and which might help me:

To find 10% → Divide by 10 → 320 ÷ 10 = 32 → 10% = £32

To find 1% → Divide by 100 → 320 ÷ 100 = 3.2 → 1% = £3.20

To find 50% → Divide by 2 → 320 ÷ 2 = 160 → 50% = £160

To find 20% → Double 10% → 32 × 2 = 64 → 20% = £64

To find 5% → Half 10% → 32 ÷ 2 = 16 → 5% = £16

To find 2.5% → Half 5% → 16 ÷ 2 = 8 → 2.5% = £8

And now we can build up our answers with a bit of simple addition!

(a) 15%

10%	£32
+ 5%	£16
15%	£48

(b) 63%

50%	£160
10%	£32
1%	£3.20
1%	£3.20
+ 1%	£3.20
63%	£201.60

(c) 17.5%

10%	£32
5%	£16
+ 2.5%	£8
17.5%	£56

(b) On a Calculator

The previous method is fine for easy numbers, and you certainly need to know how to do it for non-calculator exam papers, but if you understand the following method, then life becomes so much easier

It all comes from the fact that percentages are just decimals in disguise.
As we found out in 5. Decimals, to turn a percentage into a decimal we divide by 100.
So, to find a percentage of something:

1. Turn the percentage into a decimal (divide by 100)
2. Multiply your amount by that decimal (use your calculator!)

Example 1

Find 23% of 135g

1. Turn 23% into a decimal:

$23 \div 100 = 0.23$

2. Multiply the amount (135g) by the decimal:

$135 \times 0.23 = 31.05$g

Remember your units!

Example 2

Find 4% of £22.45

1. Turn 4% into a decimal:

$4 \div 100 = 0.04$

It's not 0.4!

2. Multiply the amount (£22.45) by the decimal:

$22.45 \times 0.04 =$ £0.90 (2dp)

Notice I have rounded my answer to 2dp as we are dealing with money

Example 3

Find 31.8% of 1,435,988

1. Turn 31.8% into a decimal:

$31.8 \div 100 = 0.318$

2. Multiply the amount (1,435,988) by the decimal:

$1{,}435{,}988 \times 0.318 = 456{,}644$
(nearest whole number)

Here I've chosen to round to the nearest whole number as the numbers were pretty big

3. Percentage Change

This is a little calculation that everyone forgets. Don't let it happen to you!

You use this when some amount has gone up or down, and you want to know by what percentage it has gone up or down by.

Formula:

$$\text{Percentage Change} = \frac{\text{New value} - \text{Old value}}{\text{Old Value}} \times 100$$

Example 1

After using mrbartonmaths.com, your mark in your maths test went from 34 to 46. What percentage increase is this?

Okay, so we just use the formula:

New Value = 46, Old Value = 34

So :

$$\text{Percentage Change} = \frac{46 - 34}{34} \times 100$$

$$= \frac{12}{34} \times 100$$

$$= 35.3\% \text{ (1dp)}$$

Example 2

What a bargain! Scientific calculators have been reduced in price from £4.99 to £3.50. What percentage decrease is this?

Again, so we just use the formula:

New Value = 3.50, Old Value = 4.99

So :

$$\text{Percentage Change} = \frac{3.50 - 4.99}{4.99} \times 100$$

$$= \frac{-1.49}{4.99} \times 100$$

$$= -29.9\% \text{ (1dp)}$$

The minus sign just means it's a decrease

4. Percentage Increase

Let me see if I can explain how to do these with a little example...

How would you find 23% of something with a calculator?... multiply by 0.23, right?
So, what would you multiply by to increase something by 23%?...
Well, it must have something to do with 0.23, but you need to add 23% onto the original.
Well, the original is 100%, and you need to add 23% onto it, which means you want... 123%
So, to increase something by 23% you need to... multiply by 1.23!

Method for Percentage Increase:
Multiply your amount by (**1** + whatever the percentage is as a decimal)

Example 1

Increase £235 by 17%

Okay, so what do we multiply 235 by?...

Well, 17% as a decimal is 0.17

So, we multiply by (1 + 0.17), which is 1.17!

235×1.17

= £274.95

Example 2

Whilst writing this website, my weight has increased by 3.5%. I used to be 87kg. What weight am I now?

Okay, so what do we multiply 87 by?...

Well, 3.5% as a decimal is... erm... 0.035

Be careful with that one!

So, we multiply by (1 + 0.035), which is 1.035!

87×1.035

= 90.045kg

5. Percentage Decrease

Again, let me see if I can explain how to do these with a little example...

How would you find 18% of something with a calculator?... multiply by 0.18, right?
So, what would you multiply by to decrease something by 18%?...
Well, it must have something to do with 0.18, but you need to subtract 18% from the original.
Well, the original is 100%, and you need to subtract 18% from it, which means you want... 82%
So, to decrease something by 18% you need to... multiply by 0.82!
Another way to think about this is once you've taken 18% away, there is only 82% left, and we all know how to find 82%, don't we?...

Method for Percentage Decrease:
Multiply your amount by (**1** - whatever the percentage is as a decimal)

Example 1

Decrease 250g by 24%

Okay, so what do we multiply 250 by?...

Well, 24% as a decimal is 0.24

So, we multiply by (1 - 0.24), which is 0.76!

Which kind of makes sense, because if you lose 24%, you are left with 76%!

250×0.76

$= 190g$

Example 2

Since I took a break from teaching, my bank balance has dropped by 64.5%. It used to be £10.20. What is it now?

Okay, so what do we multiply 10.20 by?...

Well, 64.5% as a decimal is... erm... 0.645

So, we multiply by (1 - 0.645), which is... let me get my calculator... 0.355!

10.20×0.355

$= £3.62$ (2dp)

6. Compound Interest

Everyone seems to hate compound interest, but if you understood Sections 4 and 5, then you are going to be flying! Try to follow this...

Example

The bank pays me a compound interest rate of 5% on my balance each year. At the start I have £30 in there. How much do I have after 25 years?

Now, what we definitely DO NOT do is to work out what 5% of £30 is, and the multiply this by 25!

Why?... Well, because you don't just earn 5% on the £30, you earn it on however much is in your bank at the end of each year, which is always growing!

Right, so each year my balance increases by 5%. Let's see what we've got...

End of Year **1**	→	30×1.05	→	£31.50
End of Year **2**	→	31.50×1.05	→	£33.075
End of Year **3**	→	33.075×1.05	→	£34.72875
End of Year **4**	→	34.72875×1.05	→	£36.4651975

Now, if we kept going like this, we would get the right answer, but it's going to take all day, and look at the size of the numbers! I can't be bothered writing all them down. No way.

But, think about what we are actually doing...

30×1.05... get the answer... $\times 1.05$... get the answer... $\times 1.05$... get the answer... $\times 1.05$... etc...

Which is just: $30 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times$...

How many times do we need to multiply by 1.05?... 25 times!

So, to work it out a quick way we can just do this sum: 30×1.05^{25}

And if you are good on your calculator, you should get: £101.59 (2dp)

7. Reverse Percentages

I'll get straight to the point: no-one spots these, everyone messes them up, but if you are careful, and if you try your best to follow this example, you'll be fine!

Example

For some strange reason, ever since I decorated it with "I Love Maths" stickers, my car has gone down in value by 23%. It is now worth £654.50. How much was it worth before?

Now, the key to spotting questions like this are words such as "used to", "old" and "before" – words that suggest you need to work out something that happened in the past.

Think about this: my car used to be worth a certain amount (call it w), then it went down in value by 23%, and it is now worth £654.50. I think I can write that information like this:

$$w \times 0.77 = 654.50$$

- w: The old value of the car
- $\times 0.77$: Decrease the old value by 23%
- $= 654.50$: Gives you the new value

So now all we need to do is work out the value of w!

Well, if you divide both sides of the equation by 0.77 you get… $w = \frac{654.50}{0.77}$

And so, using my calculator, w must equal… 850

So my car used to be worth £850

And the beauty of these questions is that you can check your answer by going back to the question:

The car used be worth £850, it's value fell by 23%.

Well… 850 × 0.77 = … wait for it… £654.50, which is exactly what we were hoping for!